

Kinematic and dynamic simulation of multibody systems is an important tool for analysis and design of a wide variety of systems and, similar to the finite-element method, it constitutes an integral part of Computer-Aided Engineering. While the finite-element method is essentially a "batch" process, the analysis of multibody systems can be inherently interactive. With the spectacular growth in computing power now available to individual users and with the improvements in simulation algorithms, it is becoming more and more feasible for engineers to use interactive and even real-time dynamic analyses in every day engineering calculations.

This text presents the main types of coordinates used to describe multibody systems, as well as the most common and useful methods used in kinematic and dynamic analyses. The first chapters introduce basic techniques essential to a further study of the literature. The book contains many original developments due to the authors and their research teams. The emphasis is on clarity of presentation, computational efficiency, and simplicity of the formulations. Each chapter contains many worked examples as well as problems and computer exercises. The book concludes with a compendium of recent developments in the field.

Contents:

1. Introduction and Basic Concepts.
2. Dependent Coordinates and Related Constraint Equations.
3. Kinematic Analysis.
4. Dynamic Analysis. Mass Matrices and External Forces.
5. Dynamic Analysis. Equations of Motion.
6. Static Equilibrium Position and Inverse Dynamics.
7. Numerical Integration of the Equations of Motion.
8. Improved Formulations for Real-Time Dynamics.
9. Linearized Dynamic Analysis.
10. Special Topics.
11. Forward Dynamics of Flexible Multibody Systems.
12. Inverse Dynamics of Flexible Multibodies.

MECHANICAL ENGINEERING SERIES

Javier García de Jalón
Eduardo Bayo

Kinematic and Dynamic Simulation of Multibody Systems

The Real-Time Challenge



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KINEMATIC AND DYNAMIC SIMULATION OF MULTIBODY SYSTEMS —The Real-Time Challenge—

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*The first author dedicates this book to the memory of Prof. **F. Tegerizo** († 1988) who introduced him to kinematics.*

*The second author dedicates the book to his wife **Elisabeth Anne** and children: **Carolina, Eduardo, Joseph, Annelise** and **Christopher**.*

PREFACE

The name *multibody* stands as a general term that encompasses a wide range of systems such as: mechanisms, automobiles and trucks (including steering systems, suspensions, etc.), robots, trains, industrial machinery (textile, packaging, etc.), space structures, antennas, satellites, the human body, and others.

The use of computer aided kinematic and dynamic simulation has emerged as a powerful tool for the analysis and design of multibody systems in fields such as automobile industry, aerospace, robotics, machinery, biomechanics, and others. The attention that it has received recently can be measured by the amount of computer-aided analysis programs proliferating in the market for engineering software, a phenomena similar to that produced by the finite element method in the early seventies for structural design. Efficient formulations for dynamics and reliable computational methods play a key role in achieving good simulation tools.

The purpose of this book is to describe not only the commonly used methods for multibody kinematic and dynamic simulation, but also the advanced topics and the state of the art techniques. These include numerical methods and improved dynamic formulations that allow *real-time simulation* response. The real time response in multibody simulation is a characteristic that the engineering profession is demanding more and more for analysis and design purposes. The analyst and designer are interested in visualizing a whole set of successive responses of a multibody in real time under different conditions, so as to get a clear picture of the actual performance of the system that will help them to optimize the design process.

The main features that characterize this book and distinguish it from other texts are:

- a) The use of the *natural or fully Cartesian coordinates* which allow for a simple representation of multibodies, and lead to important advantages for kinematic and dynamic simulation.
- b) The consideration of advanced topics such as: friction, backlash, forward and inverse dynamics of flexible multibodies, sensitivity analysis, and others.
- c) The detailed description of numerical methods and improved dynamic formulations that allow *real time simulation* response.

Contents

The first part of the book contains a description of the basic approaches and methods for kinematic and dynamic analyses. Chapter 1 serves as an introduction where the basic concepts and definitions are explained, the different types of problems identified, and the general ways they may be solved are outlined. Chapter 2 describes the types of coordinates commonly used for the analysis of multibody systems. Emphasis is placed on the fully Cartesian coordinates for 2- and 3-dimensional systems which are treated thoroughly along with the types of constraint conditions that they generate for different kinematic pairs. Chapter 3 deals with kinematic analysis. The solution of problems such as initial position, finite displacements, finding of the velocities and accelerations, treatment of redundant constraints (over constrained systems), and the study of the Jacobian nullspace that contains the possible motions are thoroughly exposed in this chapter. Dynamic analysis starts in Chapter 4 with the formulation of the inertia forces (mass matrices) generated by the different kind of bodies, and the external and gravitational forces. Chapter 5 continues with a detailed description of the different methods currently available for the dynamic analysis. Special attention is given to the description of the methods in both dependent and independent coordinates, and those based on velocity transformations. Chapter 6 deals with the analysis static equilibrium position and the inverse dynamic problem.

The more advanced topics are dealt with in the second part initiated in Chapter 7 which describes the numerical integration of the resultant equations of motion. Attention is given to the methods available for the solution of nonlinear ordinary differential equations and differential-algebraic equations, and emphasis is placed not only on accuracy but on stability for real time simulation. Improved dynamic formulations of order $O(N)$ and $O(N^3)$ such as recursive formalisms, improved use of velocity transformations, and some particular implementations of the penalty formulations in dependent coordinates are dealt with in Chapter 8. Emphasis is placed on the real time simulation from the viewpoint of versatility, generality, ease of implementation and possibilities of parallelization. The linearized dynamic analysis is treated in Chapter 9. Chapter 10 deals with further topics such as backlash, Coulomb friction, impacts, singular positions, kinematic synthesis, and sensitivity analysis. Some of these topics offer open areas for further research. Chapter 11 covers the forward dynamic analysis (simulation problem) of multibodies with flexible elements. The formulations that arise from the use of moving reference frames as well as inertial frames that require large displacements and rotation elastic theories are explained in detail. Chapter 12 deals with the newly developed inverse dynamics of flexible multibodies that leads to the time anticipatory joint efforts capable of reproducing a specified endpoint trajectory.

Audience for the Book

The aims of the book are twofold: educational and tutorial on the one hand, and a state of the art compilation of techniques and results on the other. The basic ideas are presented in sufficient detail in the first part of the book which can be used either as a textbook for an undergraduate elective course or basic graduate class on computer aided kinematic and dynamic analysis of mechanical systems, or as a self-study tool for the newcomer. The more advanced topics presented in the second part of the book are aimed at both the graduate student and researcher, who will find a compendium of state of the art information and a number of areas identified for further research. This part may be of interest not only to the mechanical, biomedical, or aerospace engineer, but also to people in other fields such as the numerical analyst, computer scientist, and even the software developer with interest in the computational aspects on the analysis of multibody systems.

Emphasis has been put on techniques which are basic to understanding the subject, and results included are felt to be of essentially permanent value. The authors believe that a very important feature of this book is the simplicity and easiness (rather than the sophistication) of the methods explained therein. All chapters include solved examples. Problem assignments can be found in addition at the end of each chapter of the first part of the book. Finally, the book requires only a minimal amount of background in physics and mathematics that does not exceed the basics shown by undergraduate junior students in science and engineering in both American and European universities. We have purposely tried to avoid more advanced mathematics such as tensorial calculus, dyads, and quaternions, which are used in this field but are not part of that basic background.

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